Experimental evidence of stochastic resonance without tuning due to non-Gaussian noises

F. J. Castro,[1] M. N. Kuperman,[2,*] M. Fuentes,[2] and H. S. Wio[2,†]
[1]*Grupo Fisicoquímica de Materiales, Centro Atómico Bariloche and Instituto Balseiro (CNEA and Universidad Nacional de Cuyo), 8400–San Carlos de Bariloche, Argentina*
[2]*Grupo de Fisica Estadística, Centro Atómico Bariloche and Instituto Balseiro (CNEA and Universidad Nacional de Cuyo), 8400–San Carlos de Bariloche, Argentina*
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In order to test theoretical predictions, we have studied the phenomenon of stochastic resonance in an electronic experimental system driven by white non-Gaussian noise. In agreement with the theoretical predictions our main findings are an enhancement of the sensibility of the system together with a remarkable widening of the response (robustness). This implies that even a single resonant unit can reach a marked reduction in the need for noise tuning.

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The phenomenon of *stochastic resonance* (SR) has attracted enormous interest due to both its potential technological applications for optimizing the response to weak external signals in nonlinear dynamical systems and its connection with some biological mechanisms. A recent review shows the state of the art [1]. There, a large number of applications are shown in science and technology, ranging from paleoclimatology [2] to electronic circuits [3,4], lasers [5], chemical systems [6], and the connection with some situations of biological interest (noise-induced information flow in sensory neurons in living systems, influence in ion-channel gating or in visual perception) [7]. Recent work has shown the possibility of achieving an enhancement of the system response (by means of the coupling of several SR units in what is called an *extended medium* [8–10]), or have analyzed the possibility of making the system response less dependent on fine tuning of the noise intensity [11], as well as looking at different ways to control the phenomenon [12].

It is worth remarking here that a majority of such studies on SR, with very few exceptions [13], have been done assuming that the noises were Gaussian. However, some experimental results in sensory systems, particularly for one kind of crayfish [14] as well as recent results for rat skin [15], offer strong indications that the noise source in these systems could be non-Gaussian. Some recent studies in neural networks also point in this direction [16]. Recent detailed studies on the source of fluctuations in some biological systems clearly indicate both that noise sources in general are non-Gaussian and that their distribution is bounded [17].

In recent work [18,19], a method of generating a non-Gaussian noise with a ''fine'' control on the degree of non-Gaussianity was introduced and some interesting results concerning the effect of using non-Gaussian noises in the study of SR were obtained. The results of some analytical approximations and of numerical simulations indicate a certain degree of enhancement of the system response when it departs from Gaussian behavior. However, a most remarkable finding was that the system shows a marked ''robustness'' against noise tuning. Such robustness means that the maximum of the signal-to-noise ratio (SNR) curve can flatten when departing from Gaussian behavior, implying that the system does not require fine tuning of the noise intensity in order to maximize its response to a weak external signal.

In this work we analyze the case of SR when the noise source is non-Gaussian but from an experimental point of view. We have studied an experimental setup similar to the one used in [3], but now using a non-Gaussian noise source which was built to exploit the form of noise introduced in [18,19], but for the particular case of white noise. In [18,19] a particular class of Langevin equations was studied, having non-Gaussian stationary distribution functions [20]. The work in [20] is based on the generalized thermostatistics proposed by Curado and Tsallis [21] which has been successfully applied to a wide variety of physical systems [22]. Those equations are

$$\dot{x} = f(x,t) + g(x)\,\eta(t), \tag{1}$$

$$\dot{\eta} = -\frac{1}{\tau}\frac{d}{d\eta}V_q(\eta) + \frac{1}{\tau}\xi(t), \tag{2}$$

where $\xi(t)$ is a Gaussian white noise of zero mean and correlation $\langle \xi(t)\xi(t') \rangle = 2D\,\delta(t-t')$, and $V_q(\eta)$ is given by [20] $V_q(\eta) = [1/\beta(q-1)]\ln[1+\beta(q-1)\eta^2/2]$, where $\beta = \tau/D$. The function $f(x,t)$ was derived from a potential $U(x,t)$, consisting of a double well potential and a linear term modulated by $S(t) \sim F\cos(\omega t)$ $[f(x,t) = -\partial U/\partial x = -U_0' + S(t)]$. This problem corresponds (for $\omega = 0$) to the case of diffusion in a potential $U_0(x)$, induced by η, a colored non-Gaussian noise. Clearly, when $q \to 1$ we recover the limit of η being a Gaussian colored noise (Ornstein-Uhlenbeck process). The stationary probability distribution for the random variable η and $q > 1$ is given by $P_q^{st}(\eta) = (1/Z_q)[1+\beta(q-1)\eta^2/2]^{-1/(q-1)}$, with $\eta \in (-\infty, \infty)$ and Z_q the normalization factor. When $q < 1$ the expression adopts the form

*Electronic address: kuperman@cab.cnea.gov.ar
†Electronic address: wio@cab.cnea.gov.ar

$$P_q(\eta)=\begin{cases}\frac{1}{Z_q}[1+\beta(q-1)\eta^2/2]^{-1/(q-1)} & \text{if } |\eta|<w\\ 0 & \text{otherwise,}\end{cases} \tag{3}$$

with $w^{-2}=(1-q)\beta$, and with noise intensity $D=2\beta^{-1}/(5-3q)$.

By applying a path-integral formalism to the Langevin equations given by Eqs. (1) and (2) and using an adiabatic-like elimination procedure [23] it was possible to arrive at an *effective Markovian approximation*. The specific details, not relevant here, have been shown elsewhere [18]. In the present work we have used a stationary probability distribution for the noise with the same form indicated above, and for $q<1$. To obtain a number with the mentioned probability distribution we generate two uniformly distributed random numbers $x \in (-w,w)$ and $y \in (0,P_q(0))$. x is accepted if $y<P_q(x)$.

To make the experiment we have used an electronic (analogical) circuit. We have used the well known Schmitt trigger, in an arrangement similar to [3]. Analogical approaches have been extensively used not only to study SR (see Sec. II C of [1]) but also to analyze the propagation of freezing fronts, spatiotemporal SR, irreversibility of classical fluctuations, SR in nonlinear electronic systems, resonant propagation, etc. [24].

The experimental setup was implemented with an LM741 integrated operational amplifier with positive feedback and variable resistors in a voltage divider configuration that could be used to set the threshold level of the circuit appropriately. The input and output signals were driven by a 12 bit data acquisition board (Advantech PCL-818L) connected to a PC. The frequency of the periodic signal provided to the circuit was chosen as 100 Hz or 250 Hz, with an amplitude that could be varied between 0 and $3.536V_{\mathrm{rms}}$. The threshold level of the circuit was set as $3.018V_{\mathrm{rms}}$. The optimal temporal resolution achieved with the system was 70 μs. The noises were obtained using a numerical generator as indicated above.

The set of parameters controlled during the experiments were the amplitude (V_s) and frequency (ν) of the periodic signal, the noise intensity D, and the parameter q of the noise distribution. For each selection of the parameter values we performed measurements comprising 300 000 points with a time lapse of 70 μs between two consecutive acquisitions. The phenomenon of SR was characterized by the residence time distribution [1]. In particular, we considered the area (A_ν) and the height (H_ν) of the first peak of the distribution, centered at $t=1/(2\nu)$. In Figs. 1 and 2 we display curves showing the behavior of A_ν as a function of D, for several situations.

Figure 1 shows the effect of varying the q parameter for $\nu=100$ Hz and $V_s=1.414V_{\mathrm{rms}}$. We can see that for each value of q there is a clear resonant noise intensity. As q decreases we find that the resonant noise intensity shifts to higher D values and the region where A_ν remains high is much wider (robustness). At the same time, the value of the maximum of each curve is higher. These results are in correspondence with previously reported ones obtained from numerical simulations [19]. In Fig. 2 we show the effect of changing the amplitude V_s and the frequency ν of the periodic signal. We can see that the resonant range of D becomes wider and the sensitivity of the system increases for increasing V_s. On the other hand, the effect of increasing the frequency is to shift the resonant peak toward higher values of D. The last effect is due to the existence of an experimental time lapse between consecutive measurements.

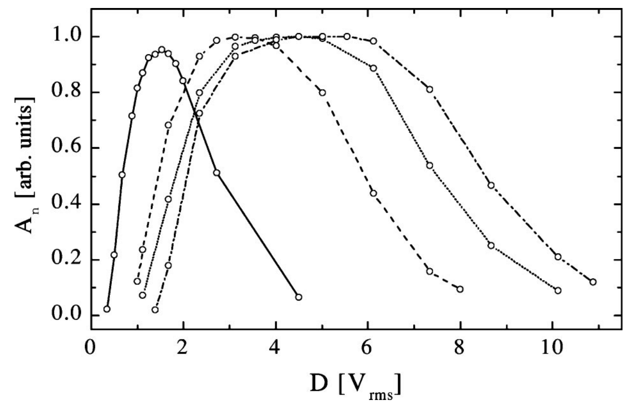

FIG. 1. A_ν vs D for several q values. $\nu=100$ Hz and $V_s=1.414V_{\mathrm{rms}}$. Solid, $q=1$; dashed, $q=0.5$; dotted, $q=0.25$; dash-dotted, $q=0.1$.

As mentioned earlier, A_ν corresponds to the area under the first peak of the residence time distribution. This quantity produces neat curves that allow us to show the SR effect more clearly. However, if we want to examine the effect of q on the sensitivity of the system it is more convenient to plot the height of the peak, H_ν, for a chosen value of V_s and ν, and several q values. This is shown in Fig. 3 where we can see that there is a clear enhancement of the sensitivity of the system with decreasing q.

Summarizing, motivated by some experimental results in sensory systems [14,15], as well as some recent theoretical results [18,19], we have experimentally analyzed the problem of SR when the noise source is non-Gaussian. We have chosen a white non-Gaussian noise source with a probability distribution based on generalized thermostatistics [21].

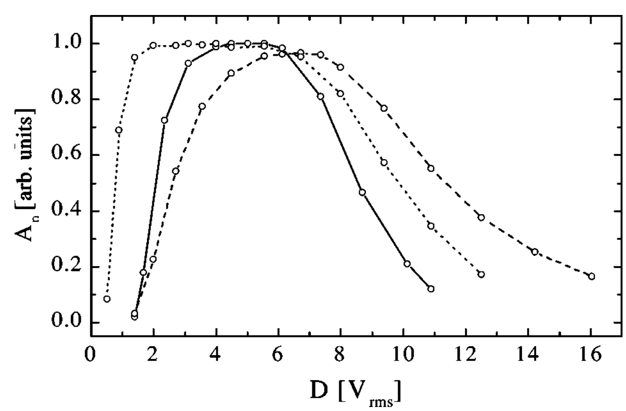

FIG. 2. A_ν vs D for $q=0.1$ and varying V_s and ν. Dotted, $\nu=100$ Hz and $V_s=2.121V_{\mathrm{rms}}$; dashed, $\nu=250$ Hz and $V_s=1.414V_{\mathrm{rms}}$; solid, $\nu=100$ Hz and $V_s=1.414V_{\mathrm{rms}}$.

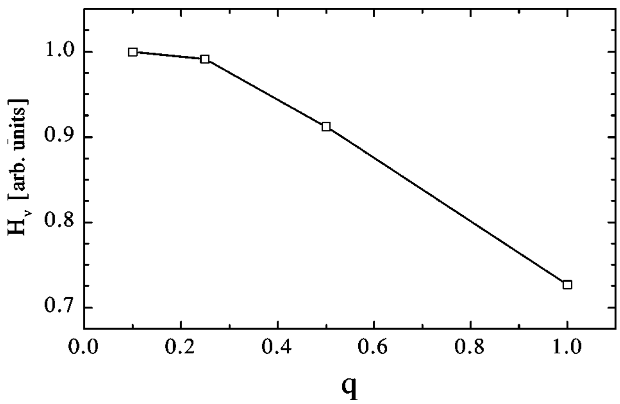

FIG. 3. H_ν vs q, for $\nu = 100$ Hz and $V_s = 1.414V_{\rm rms}$.

Our experimental results indicate the following. (1) The resonance range increases significantly with increasing non-Gaussianity of the noise. This shows that tuning the noise intensity is not necessary in order to increment the signal perception. This effect is what we call increased robustness of the system. (2) At the same time there is a clear enhancement of the sensitivity of the system (increase of H_ν) with decreasing q. (3) For larger amplitude (V_s) of the periodic signal, the ''resonant'' range for D becomes wider and the height of the response increases. (4) The effect of increasing the frequency ν is to shift the resonance peak toward higher values of D.

As we depart from Gaussian behavior (with $q < 1$), the SNR shows two main aspects: first it becomes less dependent on the precise value of the noise intensity, and secondly its maximum as a function of the noise intensity increases. Both aspects are of great relevance for technological applications [1]. Moreover, as was indicated in [15], non-Gaussian noises could be an intrinsic characteristic in biological systems, particularly in sensory systems [7,14,15]. In addition to the increase in the response (SNR), the reduction in the need for tuning a precise value of the noise intensity is of particular relevance both in technology and in order to understand how a biological system can exploit this phenomenon. It is worth remarking here that such effects have been obtained considering a single resonant unit, and not through the coupling of several resonant units as usual. According to the results of previous work [8–10], one should expect an even larger effect when several units are coupled. Here we have focused on the case of white non-Gaussian noise; the case of colored non-Gaussian noise will be the subject of a forthcoming work.

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